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The width of the Θ^+ exotic baryon in the chiral soliton model

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Abstract. In 1997 Diakonov, Petrov, and Polyakov, calculated the width of the exotic baryon that they called Θ^+. The prediction, $\Gamma(\Theta^+) \lesssim 15\,\mathrm{MeV}$, has received considerable attention, especially in light of the narrowness of the experimentally reported Θ^+ resonance. However, there is an arithmetic error in their work: when corrected, the width estimate quoted in that paper should have been $\lesssim 30\,\mathrm{MeV}$.

The existence of a relatively light exotic antidecuplet of baryons was first pointed out in the context of the Skyrme model by Manohar in 1984 [1–3][1] The mass of the lightest and potentially most prominent member of the antidecuplet, now known as the Θ^+, was computed by Praszalowicz in 1987 [7]. He predicted that this $I = 0$, $Y = 2$ K^+n resonance would have a mass of approximately 1530 MeV.

In a 1997 paper remarkable for its foresight, Diakonov, Petrov and Polyakov studied the antidecuplet in the chiral soliton model [4]. They obtained a mass estimate close to Praszalowicz's and observed that experiments had not yet probed this region thoroughly. Their paper stimulated an experimental search for the Θ^+. They also presented qualitative arguments that the Θ^+ could be quite narrow. It is clear from their paper that they believed this to be independent of any specific model calculation. However, they present, and quote in the abstract and conclusions, a specific calculation of the width of the Θ^+ and other antidecuplet baryons. However, the 15 MeV width they quote for the Θ^+ cannot be obtained from their equations. Instead, evaluation of their equations yield $\Gamma(\Theta^+ \to NK) \approx 30\,\mathrm{MeV}$. The error is arithmetic. Their model is simple, and clearly and consistently presented. One parameter $(G_0 + \frac{1}{2}G_1)$ is fit to the width of the $\Delta(1232)$, the other (G_1/G_0) is taken from chiral quark soliton models. The results do not follow from the numbers.

The purpose of this short note is to correct the arithmetic in [4]. Normally this would not require publication. However, three considerations motivate broad distribution of this comment: First, of course, the Θ^+ has been discovered and appears to be very narrow [8]; second, the "prediction" of $\Gamma(\Theta^+) \approx 15\,\mathrm{MeV}$ in [4] is frequently and prominently cited as an explanation of the observed width; and finally, the authors of [4] have declined to provide an erratum to their paper that would correct the misperception propagated by this error.[2]

There is no physics at issue, so it is not necessary to review the authors' model or the method by which they compute widths. The issue is arithmetic. Reference [4] presents separate expressions for the partial widths for $\Delta \to N\pi$ (see (42)), $\Sigma^* \to \Lambda\pi$ (see (43)), $\Sigma^* \to \Sigma\pi$ (see (44)), $\Xi^* \to \Xi\pi$ (see (45)), and $\Theta^+ \to NK$ (see (56)), and for other antidecuplet decays (see (57)–(67)). All of these equations are consistent with and summarized by their equation (49), "the ... general formula for partial widths of members of the decuplet and of the antidecuplet" [4],

$$\Gamma(B_1 \to B_2 M) = \frac{3G_r^2}{2\pi(M_1 + M_2)^2}|\mathbf{p}|^3 \frac{M_2}{M_1}\left(C_1 + \frac{1}{\sqrt{5}}C_2 c_{\overline{10}}\right). \quad (1)$$

Here M_1 and M_2 are the masses of baryons B_1 and B_2, respectively. $|\mathbf{p}|$ is the center of mass momentum in the decay,

$$|\mathbf{p}| = \frac{1}{2M_1}\sqrt{M_1^4 + M_2^4 + m^4 - 2m^2M_1^2 - 2M_1^2M_2^2 - 2M_2^2m^2}.$$

m is the meson mass. G_r is a sum of Yukawa coupling constants which differs for an initial decuplet, $r = 10$, or antidecuplet, $r = \overline{10}$,

$$G_{10} = G_0 + \frac{1}{2}G_1,$$
$$G_{\overline{10}} = G_0 - G_1 - \frac{1}{2}G_2, \quad (2)$$

as listed in Table 2 of [4]. The authors take $G_2 \approx 0$ and $c_{\overline{10}} \approx 0$ in their numerical evaluation of widths. The constant C_1

[1] Diakonov, Petrov and Polyakov [4] cite [5] in this regard. However, there is no mention of the antidecuplet in either of the quoted papers [6].

[2] The error in [4] appears to have been recognized immediately after publication of [4] by Weigel in [9]. He discusses the issue in a footnote on page 17 of the e-print.

is a Clebsch–Gordan coefficient. It equals 1/5 for both $\Delta \to N\pi$ and $\Theta^+ \to NK$, which are the decays relevant here. To compute the decuplet and antidecuplet widths, it is necessary to know both G_0 and G_1. The authors fit $G_0 + \frac{1}{2}G_1$ to the width of the $\Delta(1232)$ and use the chiral quark soliton model to estimate $G_1/G_0 \approx 0.4$.

The crux of the issue is the application of (1) to the Δ and Θ^+ decays. The authors provide the formulas for each case. Equation (42) applies to the Δ,

$$\Gamma(\Delta \to N\pi) = \frac{3(G_0 + \frac{1}{2}G_1)^2}{2\pi(M_\Delta + M_N)^2}|\mathbf{p}|^3 \frac{M_N}{M_\Delta}\frac{1}{5}, \qquad (3)$$

where, following [4] I have replaced G_0 by $G_0 + \frac{1}{2}G_1$. Equation (56) applies to the Θ,

$$\Gamma(\Theta \to NK) = \frac{3(G_0 - G_1)^2}{2\pi(M_\Theta + M_N)^2}|\mathbf{p}|^3 \frac{M_N}{M_\Theta}\frac{1}{5}, \qquad (4)$$

where, following [4] I have dropped a term proportional to G_2 and another one proportional to $c_{\overline{10}}$. Substituting known masses and $\Gamma(\Delta \to N\pi) = 110\,\mathrm{MeV}$ into (3), one finds

$$G_0 + \frac{1}{2}G_1 \approx 25, \qquad (5)$$

in disagreement with the value $G_0 + \frac{1}{2}G_1 \approx 19$ quoted in (54) of [4]. This appears to be an arithmetic error.[3] Substitution of the correct value, (5), into (4) along with $G_1/G_0 \approx 0.4$ [4] gives

$$\Gamma(\Theta^+ \to NK) \approx 30\,\mathrm{MeV}, \qquad (6)$$

in disagreement with the result $\Gamma(\Theta^+ \to NK) = 15\,\mathrm{MeV}$ claimed in [4].

It is also worth noting that the arithmetic error afflicts the value of the πN coupling constant, $g_{\pi NN}$, in the model of [4]. Instead of $g_{\pi NN} \approx 13.3$ as claimed in (54) of [4], the corrected result is

$$g_{\pi NN} = \frac{7}{10}\left(G_0 + \frac{1}{2}G_1\right) \approx 17.5,$$

to be compared with the experimental value of ≈ 13.6. If $G_0 + \frac{1}{2}G_1$ is adjusted to obtain the correct value of $g_{\pi NN}$, then the model prediction of the width of the Δ is too small by almost a factor of two. The arithmetic error generates minor corrections to the widths of the decuplet baryons listed in (42)–(45). It also modifies the predicted widths of the other antidecuplet states, increasing all of them by roughly a factor of two.

To summarize, the purpose of this short note has been to point out and correct an arithmetic error in [4]. Since arithmetic is performed according to universal rules, and is considerably simpler than theoretical physics, the reader should feel free to check the issue for him or herself directly.

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[3] In [9] Weigel pointed out that the discrepancy would be explained if the authors of [4] had mistakenly used M_Δ/M_N in place of M_N/M_Δ in (3). This error was subsequently acknowledged in correspondence between Weigel and one of the authors of [4, 10]